particle thermal relaxation time; E, activation energy; R, universal gas constant; U, potential; $\delta(x)$, delta-function; q, heterogeneous reaction rate; π, split probability. Subscripts: o, initial state; 1, 2, parameters of two stable states; p, particle.

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PERCOLATION AND DIFFUSION IN FRACTAL TURBULENCE

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Experimental data on passive-impurity diffusion in fractal turbulence are interpreted on the basis of a percolational model.

INTRODUCTION

The geometry of the eddy field in turbulized fluid is extremely fragmentary. Fragments of fluid with high vorticity are interspersed with fragments with low vorticity. Taking into account that in steady turbulent motion there is practically no transfer of passive impurity from the turbulent fragments to laminar fragments, but simply motion of the impurity with (and within) the turbulent fragments, fractal theory may be used to describe turbulent diffusion [1, 2]. The fact is that the problem of passive-impurity diffusion in homogeneous turbulence is still far from solution. Experiments with hydrodynamic lattices to model homogeneous turbulence give inconsistent data, and remain to be interpreted [3, 4]. In large-scale experiments in the ocean, which may also model homogeneous turbulence, the well-known Richardson 4/3 law for the effective diffusion coefficient is also found to be nonuniversal [5]. The reason for this is unclear. Could it be that the assumption of statistical homogeneity [6] is too limiting? Recently, this hypothesis has been weakened, substituting the less restrictive requirement of geometric self-similarity within some range of scales (e.g., see [1, 2]). This fractal approach may allow some of the features of impurity transport noted above to be taken into account and yield a theory which approximates the experimental effects.

Several fractal models of turbulence now exist (e.g., see [1, 2, 7]) and more will certainly appear in the future, since the approach to self-similar motion may take different forms, converging asymptotically on similar quasi-stable states [8]. The choice of a particular model of this process is largely dictated by considerations of convenience. In the

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present work, the fractal model adopted is the statistical model of critical percolation (e.g., see [9]), which is the most convenient for the investigation of passive-impurity diffusion.

1. PERCOLATIONAL MODEL OF TURBULENT MOTION

For the percolational model, the region in which the fluid moves is notionally divided into cubic cells of side η (the Kolmogorov scale [6]).

Eddies with this scale dissipate rapidly under the action of viscosity. At any given moment, the motion is turbulent in some η cells and laminar in others. This is known as intermittency. The probability p of turbulization of each cell may now be introduced: when $p = 0$, there are no turbulized cells; when $p \ll 1$, there are clusters consisting of a few turbulized cells; when $p = 1$, all the cells are turbulized. There is a critical concentration $0 < p_0 < 1$ at which an infinite cluster of turbulized cells initially appears. The appearance of this cluster significantly changes the physical situation. Whereas increase in the number of turbulized cells and dissipative processes account for the energy introduced before the appearance of this cluster, the presence of the cluster enables the energy to be dissipated to infinity, i.e., removed from the region of motion. In the presence of an infinite cluster, the concentration of turbulized cells may fluctuate. However, if there is balance between the energy introduced and the energy withdrawn through the cluster, the supercritical concentration appearing fluctuationally is removed because of the lack of an energy supply and the action of viscosity. In addition, only the skeleton is stable within the infinite cluster itself (for the same reason) [9]. The skeleton of the cluster is a set of cells belonging to infinite paths through the cluster, i.e., dead-end (finite) cell branches are damped under the action of viscosity for want of an energy input.

This is a percolational lattice model; (e.g., see [9]). The clusters forming in these models are fractals [9], and the property of self-similarity replaces the concept of homogeneity. In this sense, the term fractal turbulence may be used instead of homogeneous turbulence. Percolational clusters are characterized by some universal parameters, one of which is the fractal dimensionality. The fractal dimensionality of a critical percolational cluster is denoted by D_i here, and that of its skeleton by D_s. It is known that $D_i \approx 5/2$ in three-dimensional space, and $D_i \approx 2.9$ in two-dimensional space [9]. In three-dimensional space, as in two-dimensional space, $D_s \approx 5/3$ to within ~10% [9]. In this context, it is appropriate to note the relation between the stable fractal dimensionality of turbulence and the index in the spectral law for the turbulent energy in the inertial interval in three-dimensional turbulence [2]

$$E(k) \sim k^{-D_s} \tag{1}$$

and to recall the Kolmogorov–Obukhov 5/3 law.

2. INTERNAL AND EXTERNAL MOTION OF PASSIVE-IMPURITY PARTICLES

The starting point here is that the passive-impurity particle, moving over the fractal volume occupied by the turbulized fluid, is practically unable to leave this volume and enter the laminar fluid. If a point is moving randomly in a homogeneous medium, the dependence of the mean square distance covered by the point in time t is

$$\langle r^2 \rangle \sim t \tag{2}$$

(Brownian motion). In motion within a fractal

$$\langle r^2 \rangle \sim t^{2/(2+\Theta)}, \tag{3}$$

where Θ is the anomalous-diffusion coefficient [9].

Equation (3) may be regarded as describing diffusion in a medium with an effective diffusion coefficient [9]

$$K_*(r) \sim r^{-\Theta}, \tag{4}$$

i.e., in motion within a fractal, the effective diffusion coefficient depends on the distance, and Eq. (4) holds for any initial point of motion. The parameter D_w, which is the internal dimensionality of random motion over the fractal, is also used here; for free uniform Brownian motion, $D_w = 2$; for motion over a fractal

$$D_w = 2 + \Theta. \tag{5}$$

The relation $D_w \geq 2$ for motion over a fractal is associated with the repeated turning of a point moving in a fractal. Knowing the dimensionality of the fractal D, may the dimensionality of internal motion be found?

According to the Alexander–Orbach hypothesis [9]

$$2D/D_w = 4/3. \tag{6}$$

The universality of this relation is dubious [9].

To establish a relation between D and D_w (and hence Θ) when D is not too large (as shown below, for $D < 3$, which is quite convenient), consider the Brownian motion of a particle of a fractal of dimensionality D in a space of dimensionality d, where d is not necessarily an integer. The fractal dimensionality D_{in} of intersection of the Brownian-motion fractal (dimensionality $D_0 = 2$) with the topological boundary of the basic fractal of dimensionality $(D - 1)$ in a space of dimensionality d is

$$D_i = D_0 + (D-1) - d. \tag{7}$$

The minimum value of d at which $D_{in} = 0$ is the fractal dimensionality of internal motion, that is

$$D_w = d_{\min} = D_0 + (D-1). \tag{8}$$

If the free motion is Brownian, i.e., $D_0 = 2$, it follows from Eq. (8) that

$$D_w = D + 1. \tag{9}$$

The fractal dimensionality of the topological boundary must be less than the fractal dimensionality of free motion here, or else the moving particle may simply be lost at the boundary. The corresponding condition is

$$D_0 > D - 1. \tag{10}$$

For Brownian free motion, $D_0 = 2$; hence Eq. (9) is applicable provided $D < 3$. Remember that $D_i \approx 5/2$ for a critical percolational cluster in three-dimensional space, i.e., Eq. (9) is applicable.

Now consider the external motion of a passive-impurity particle. If the fractal dimensionality of the topological fractal boundary is less than that of the free-particle motion, i.e., Eq. (10) holds, the fractal dimensionality of the external impurity-particle motion will simply coincide with that of the topological boundary (since the particle cannot leave the fractal). Denoting the fractal dimensionality of external motion by D'_w, it follows that if Eq. (10) holds,

$$D'_w = D - 1. \tag{11}$$

To elucidate the role played by the topological boundary of the fractal, which is of dimensionality $(D - 1)$ and determines the fractal dimensionality of internal and external motion, consider a sphere of radius R. The number N of η cells within this sphere will increase with R as follows

$$N \sim R^D, \tag{12}$$

and here the impurity-particle flux over the fractal through the surface of this sphere is determined by the relative area intersected by the fractal at its surface, which increases with increase in R as R^{D-1}.

3. CONCENTRATION OF PASSIVE IMPURITY

The equation for the passive-impurity concentration at the fractal may be written in spherical coordinates as follows [9]

$$\partial c/\partial t = \frac{1}{r^{D-1}} \frac{\partial}{\partial r} K r^{D-1-\Theta} \frac{\partial c}{\partial r} - (u\nabla) c, \tag{13}$$

where K is the diffusion coefficient; the second term on the right-hand side is the convective impurity transfer. The variation in the first (diffusional) term on the right-hand side is related to the fractal properties of the region of impurity motion (Sec. 2) and the effective diffusion coefficient: $K(r) = Kr^{-\Theta}$.

When r is sufficiently small and $\Theta > 0$, the second term in Eq. (13) may be neglected

in comparison with the first, and the role of turbulence in impurity transport in this case reduces to the creation of the fractal region of its motion, i.e., to the formation of the effective diffusion coefficient.

At the asymptote of large r, the solution of Eq. (13) (without the last term) takes the form [9]

$$c(t) \sim (Kt)^{-D/D_w}. \tag{14}$$

As follows from Eq. (14), $m = 2D/D_w$ is the index in the asymptotic degeneracy law $c^2(t)$; in percolation theory, it is called the fraction dimensionality [9]. The Alexander–Orbach hypothesis mentioned above, according to which $m = 4/3$, remains hypothetical in percolational theory, but is used as an approximation.

In laboratory experiments on the degeneracy of passive impurity behind hydrodynamic lattices, various values of m are obtained. For example, in [3], generalizing data for m obtained in different experiments, $m = 1.31 \pm 10\%$ was proposed. This conclusion obviously conflicts with the Alexander–Orbach hypothesis. However, the fairly significant spread in the values of m from experiment to experiment necessitates some explanation of the nonuniversality of the values observed for m.

Turning to the results of Sec. 2, this nonuniversality may be attributed to variation in D (and also D_w) in the different experiments. The variation in D is due to the instability of D_i (Sec. 1), which relaxes to D_s under the action of viscosity; therefore, D varies in the range

$$D_s \leqslant D \leqslant D_i. \tag{15}$$

In Sec. 2, Eq. (9) expresses the relation between D and D_w when $D < 3$. The range of variation of the index m

$$c^2(t) \sim t^{-m}, \tag{16}$$

follows from Eqs. (15) and (9)

$$10/8 \leqslant m \leqslant 10/7 \tag{17}$$

(for three-dimensional turbulence).

The variability in m in the known experimental data for hydrodynamic lattices agrees with Eq. (17).

4. EXTERNAL DIFFUSION

Whereas the degeneracy of the passive-impurity concentration is determined by the internal motion of the impurity particle, effects such as impurity-cloud expansion are determined by the external motion. These and other properties depend on the fractal dimensionality of turbulence. When the fractal dimensionality of the topological boundary of turbulence is less than that of the trajectories of free motion of impurity particles, the external motion is determined by the fractal dimensionality of this boundary (D − 1) (Sec. 2). In this case, D_w in Eq. (5) must be replaced by D_w' to find Θ. Then

$$\Theta = D_w' - 2 = D - 3. \tag{18}$$

In contrast to internal motion, $\Theta < 0$ for external motion.*

Taking account of Eq. (18), it follows from Eq. (4) for the effective diffusion coefficient that

$$K_* \sim r^{3-D}. \tag{19}$$

For three-dimensional turbulence, it follows from Eq. (19) that in the case of unstable fractal dimensionality $D_i \approx 5/2$ (Sec. 1)

*Note that the dynamics of turbulent-cluster transformation impose known limits on the applicability of the given model. Thus, the characteristic time of turbulization (or degeneracy) of the cell with constant variation in cluster configuration must be considerably less than the characteristic times of the diffusion processes. Quantitative estimates of these time intervals fall far beyond the scope of the present. It is simply noted here that the situation improves with increase in Reynolds number.

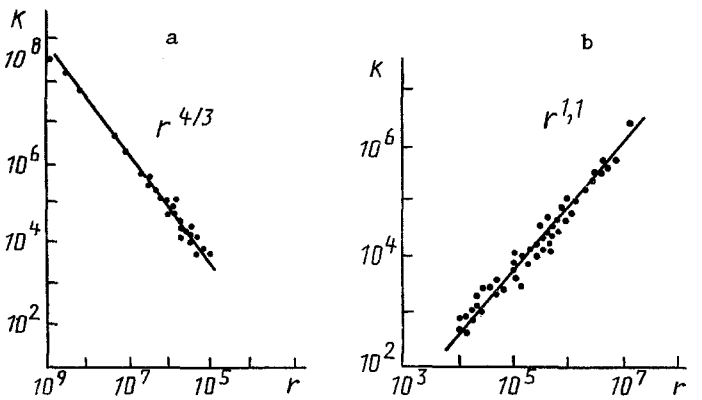

Fig. 1. Experimental observation of stable (a) and unstable (b) fractal dimensionality in a quasi-two-dimensional situation, K, cm^2/sec; r, cm.

$$K_* \sim r^{1/2}, \tag{20}$$

and for stable fractal dimensionality $D_s \approx 5/3$

$$K_* \sim r^{4/3}, \tag{21}$$

i.e., the well-known Richardson law [5, 6].

For oceanological experiments, the two-dimensional case with impurity diffusion is of great interest. In this case, $D_i \approx 1.9$, and hence

$$K_* \sim r^{1/1}, \tag{22}$$

while for $D_s \approx 5/3$

$$K_* \sim r^{4/3}. \tag{23}$$

Laws of the type in Eqs. (22) and (23) are typical for oceanological experiments on impurity diffusion; see [5], for example. Two types of dependence of K_* on r observed in oceanological experiments are shown in Fig. 1, which is taken from [5]. The realization of a particular type corresponds to the stable – Eq. (23) – or unstable – Eq. (22) – situation in the given case.

It is interesting to ask why both the stable and unstable states may be realized in oceanological experiments, whereas only the stable situation in Eq. (21) is evidently realized in atmospheric experiments. Note that, as follows from the foregoing, both stable and unstable fractal dimensionalities are evidently realized in experiments behind hydrodynamic lattices.

CONCLUSIONS

Thus, very complex and diverse geometric structures are present in turbulence which appears homogeneous at first glance. These structures have the properties of spatial self-similarity, which may be replaced by the property of homogeneity in their description. Universal critical characteristics of these structures of the type of fractal dimensionality have a significant influence on the impurity transport in such turbulence. Taking this influence into account allows experimental data on impurity transport in laboratory conditions to be related to data obtained in large-scale natural processes.

NOTATION

η, cosmological length scale; D, fractal dimensionality; c, passive-impurity concentration; K, diffusion coefficient.

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BOUNDARY CONDITIONS FOR THE HEAT- AND MASS-TRANSFER EQUATIONS OF COARSELY DISPERSE AEROSOLS IN A TURBULENT FLOW

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Boundary conditions taking account of particle interaction with the boundary surface are obtained on the basis of the Chapman–Enskog method of solving the kinetic equation.

In the turbulent flow of coarsely disperse aerosols in channels, the dynamic relaxation time of the particles considerably exceeds the lifetime of the energy-content pulsations of the carrying flux. In this case, processes occurring in the interaction of the discrete phase with the surface exert a significant influence on the dynamic and thermal characteristics of the disperse flow. In calculations of the turbulent disperse flows, the collision of particles with the walls is taken into account by formulating the corresponding boundary conditions for the hydrodynamic and heat- and mass-transfer equations. In [1-4], the boundary condition for the concentration of Brownian particles at a partially absorbing surface was constructed. The boundary concentration for the concentration of disperse impurity in the turbulent flow, taking account of inhomogeneity of the turbulent-pulsation field, the mass force, and the degree of particle entrainment in the turbulent motion, was found in [5]. The boundary conditions of [1-5] are conditions of the third kind for the particle-diffusion equations and relate the concentration value and its gradient at the surface. The distinguishing feature of these boundary conditions is the nonzero particle concentration at an absolutely absorbing wall.

The distribution of the pulsational characteristics of inertial particles in an inhomogeneous turbulent flow is determined by the ratio between the scale of inhomogeneity of the pulsational field of the carrier phase and the pulsational inertial path length of the particle $\ell_p = \tau_u \sigma^{1/2}$ [6, 7]. If the inertial path length of the particle is comparable with the characteristic scale of inhomogeneity of the turbulent pulsations of the fluid phase close to the channel wall, the intensity of pulsational motion of the discrete phase in the wall region is determined by the pulsational energy of the particle acquired in the flow core. In this case, the intense turbulent motion of particles around the wall leads to effective turbulent transfer of the mean flow characteristics (axial velocity, temperature) from the flow core to the surface. The description of such flow must be based on the two-velocity and two-temperature approximation, when the equations for the particle velocity and temperature are used together with the equations for the mean velocity and temperature of the carrier phase. On account of the intense transverse turbulent transfer in the solid phase between the flow core and the wall region, the equations for the mean characteristics of the discrete phase are of diffusional type. Accordingly, the formulation of boundary conditions taking account of the processes occurring at particle contact with surface is an urgent problem.

In the present work, a closed system of equations and boundary conditions for calculat-

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